OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 16 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			**DR** $y = 4 - 2x$				
			$4 - 2x = x^2 + x$	**M1**	**2.1**	Eliminating x or y must be seen	
			$\Rightarrow x^2 + 3x - 4 = 0$	**M1**	**1.1**	Form a quadratic equation	Or $y^2 - 14y + 24 = 0$
			$\Rightarrow x = 1$ or $x = -4$	**A1**	**1.1**		SC1 for one pair of coordinates only
			$y = 2$ or $y = 12$	**A1**	**1.1**		
			(1,2) and (-4,12)	**A1**	**2.5**	For final A mark, corresponding values of x and y must be expressed as coordinates from well set out correct solution	
				[5]			
2			[1-way] stretch	**M1**	**1.1**		
			scale factor 2 in y-direction	**A1**	**1.1**		
			translation	**M1**	**1.1**	If transformations given in reverse order then **M1**, **A1**, **M1** are still available (but not final **A1**)	
			$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	**A1**	**1.1**	Or -1 in y-direction	
				[4]			

Question			Answer	Marks	AOs	Guidance	
3			$\int_0^{\frac{\pi}{12}} \cos 3x \mathrm{d}x = \left[\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{12}}$	B1	1.1	$\frac{\sin 3x}{3}$	
			$= \frac{1}{3}\left(\sin\frac{\pi}{4} - 0\right)$	M1	1.1		
			$= \frac{\sqrt{2}}{6}$ o.e.	A1 [3]	1.1	Must be in exact form	
4			$y = x^3 - 4 \;\; x \leftrightarrow y$ $x = y^3 - 4$ $\Rightarrow x + 4 = y^3$	M1	1.1	attempt to invert	
			$\Rightarrow y = \sqrt[3]{x+4}$ so $f^{-1}(x) = \sqrt[3]{x+4}$	A1	1.1	accept $y = \sqrt[3]{x+4}$ but not $x = \sqrt[3]{y+4}$	
			range of f is $-5 \leq y \leq 4$	M1	1.1	May be implied	
			so domain of f^{-1} is $-5 \leq x \leq 4$	A1	1.2	or $[-5, 4]$	
			range is $-1 \leq y \leq 2$	B1 [5]	1.1	or $-1 \leq f^{-1}(x) \leq 2$ or $[-1, 2]$	
5			Binomial(20, 0.08) P(2 blue) = 0.27[11]	M1 A1 [2]	3.3 1.1	**BC**	
6	(i)	(*A*)	Mean = 17	B1	3.4		
		(*B*)	**Either** Points of inflection are approx. 3 above and below mean so SD = approx. 3	E1	2.4	AG	
			Or Limits are approx. 9 above and below mean so SD = 9 ÷ 3 = 3	E1	2.4	AG	
				[2]			

Question			Answer	Marks	AOs	Guidance	
6	(ii)		Mean in Fahrenheit = 1.8 × 17 + 32 = 62.6 SD in Fahrenheit = 1.8 × 3 = 5.4	**B1** **B1** **[2]**	**1.1** **1.1**	FT their mean	
7			$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ = 0.6 + 0.5 – 0.85 = 0.25 $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.25}{0.5}$ = 0.5	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **1.1** **1.1** **1.1**		
8			Increases a value by 6 New value is closer to 62 than the old value is to 61.4 51 changes to 57 **or** 57 changes to 63 **or** 58 changes to 64 **[3]**	**M1** **M1** **A1**	**3.1b** **2.2a** **2.2a**	Implied by correct answer or pair of values differing by 6 Implied by correct answer or new value closer to 62 than old value	
9	(i)		0.758 > 0.279 So there is sufficient evidence of correlation (in the population) **[2]**	**M1** **A1**	**1.1** **2.2b**	Oe but not evidence of positive correlation.	
9	(ii)		E.g. diagram shows positive correlation overall, but the data consists of two distinct clusters. E.g. neither of the two clusters show evidence of correlation **[2]**	**B1** **B1**	**2.3** **2.2b**	Accept other suitable correct comments	

Question			Answer	Marks	AOs	Guidance	
10	(*A*)		E.g. Will not sample people who work then/people who do not walk down that street.	**B1**	**2.4**		
	(*B*)		E.g. This will only get answers from those who want to send in an answer.	**B1**	**2.4**		
	(*C*)		E.g. This will only get answers from those who use the council website. E.g. Those who use the internet more frequently are more likely to see the question.	**B1** **[3]**	**2.4**		
11			Suppose $x + y$ is rational	**E1**	**2.1**		
			So $x + y = \frac{p}{q}$, where p and q are integers	**B1**	**2.1**		
			$\Rightarrow x = \frac{p}{q} - \frac{m}{n} = \frac{(pn - mq)}{qn}$ which is rational	**B1**	**3.1a**	or stating that the difference of two fractions is rational	
			x is irrational so this is a contradiction	**E1** **[4]**	**2.4**		
12	**(i)**		$6x^2 + 3y^2 \frac{dy}{dx} = 5\frac{dy}{dx} \left[\Rightarrow \frac{dy}{dx} = \frac{6x^2}{5 - 3y^2}\right]$	**M1** **A1**	**1.1a** **1.1**	implicit differentation correct	
			when $x = 1$, $y = 2$, $6 + 12\frac{dy}{dx} = 5\frac{dy}{dx}$	**M1**	**1.1**	substituting $x = 1$, $y = 2$	
			$\Rightarrow \frac{dy}{dx} = -\frac{6}{7}$	**A1** **[4]**	**2.1**	cao	
12	**(ii)**		$\frac{dy}{dx} = 0$ so $6x^2 = 0$	**B1**	**1.2**	Substitute $\frac{dy}{dx} = 0$ into their differentiated expression	
			$x = 0$ so all stationary points lie on y-axis	**E1** **[2]**	**2.1**	Completion of argument	

Question			Answer	Marks	AOs	Guidance	
13			let $u = 1+\sqrt{x}$ $\quad du = \frac{1}{2}x^{-\frac{1}{2}}\,dx$	M1	3.1a	substituting $u = 1+\sqrt{x}$ or $w = \sqrt{x}$	Evidence of method must be seen
			$\Rightarrow dx = 2(u-1)du$	A1	1.1	$dx = 2(u-1)\,du$ or $dx = 2w\,dw$	
			$\Rightarrow \int_0^1 \frac{1}{1+\sqrt{x}}dx = \int_1^2 \frac{2(u-1)}{u}du$	A1	1.1	$\frac{2(u-1)}{u}(du)$ or $\frac{2w}{(w+1)}(dw)$	
			$= \int_1^2 \left(2-\frac{2}{u}\right)du$	M1	3.1a	splitting fraction or dividing to get $2 - \frac{2}{(w+1)}$ (or substituting $u = w+1 \Rightarrow \frac{2(u-1)}{u}$ and then splitting fraction)	Evidence of method must be seen
			$= \left[2u - 2\ln u\right]_1^2$	A1	1.1	or $\left[2w - 2\ln(w+1)\right]_0^1$ if still in terms of w	
			$= 4 - 2\ln 2 - 2 = 2 - 2\ln 2$ **or** $2 - \ln 4$	A1	1.1	cao	
				[6]			

Question			Answer	Marks	AOs	Guidance	
14	(i)		$\int \frac{dm}{m} = \int \frac{dt}{t(1+2t)}$	M1	1.1a	separating variables	
			$\frac{1}{t(1+2t)} \equiv \frac{A}{t} + \frac{B}{1+2t}$	M1	3.1b	using partial fractions	
			$\Rightarrow 1 \equiv A(1 + 2t) + Bt$ $t = 0 \Rightarrow A = 1$	M1	1.1	substituting values, equating coeffs or cover up	
			$t = -½ \Rightarrow 1 = -½ B \Rightarrow B = -2$	A1A1 1.1	1.1	$A = 1$, $B = -2$	
			$\Rightarrow \int \frac{dm}{m} = \int \left(\frac{1}{t} - \frac{2}{1+2t}\right) dt$ $\Rightarrow \ln m = \ln t - \ln(1 + 2t) + c$	B1FT	2.1	FT their A, B, condone no c	
			$t = 1$, $m = 1 \Rightarrow c = \ln 3$	M1	1.1	evaluating constant of integration	
			$\Rightarrow \ln m = \ln\left(\frac{3t}{1+2t}\right)$	E1	2.1		
			$\Rightarrow m = \frac{3t}{(1 + 2t)}$			AG	
				[8]			
14	(ii)	(*A*)	$1.25 = \frac{3t}{(1+2t)}$	M1	1.1a		
			$\Rightarrow 1.25 + 2.5t = 3t$ $\Rightarrow t = 1.25 \div 0.5 = 2.5$ minutes	A1	1.1		
				[2]			
14	(ii)	(*B*)	$m = \frac{3}{\left(\frac{1}{t} + 2\right)}$	M1	3.1b	or substituting a large value for t	
			$\rightarrow 1.5$ [grams]	A1	2.2a		
				[2]			

Question			Answer	Marks	AOs	Guidance	
15	(i)		Estimated number $= 4 + \frac{16}{3} = 9\frac{1}{3}$ $\frac{9\frac{1}{3}}{80} = 0.1166...$ so proportion is approximately 0.117	**M1** **A1** **[2]**	**3.1b** **1.1**	for attempt at interpolation	
15	(ii)		E.g. Midpoints Mean = 170 Standard deviation = 3.4	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	evidence of valid method for estimation **BC** Mean in the range 169-171 **BC** SD in the range 3-3.5	
15	(iii)		The histogram e.g. seems to have a rough bell shape e.g. is symmetrical (around the estimated mean) e.g. appears to have all data within 3 s.d. of the mean so this does support the manager's belief	**B1** **B1** **[2]**	**3.5a** **3.5a**	for one reason for at least two reasons and 'supports belief'	
15	(iv)	(*A*)	P(Lifetime > 174) for N(170, 3.4^2)	**M1**	**3.4**	oe	
			0.1197	**A1**	**1.1**	**BC** FT their mean and standard deviation	
		(*B*)	Answer is very similar to estimate in part (i)	**B1** **[3]**	**3.5a**		

Question			Answer	Marks	AOs	Guidance	
15	(v)		**Either** Test statistic = $\frac{207.3-210}{3.4/\sqrt{8}} = -2.246$	**M1**	**3.4**	Must include √8	
			Lower 5% level 1 tailed critical value of $z = -1.645$	**A1**	**1.1**		
			−2.246 < -1.645 so significant	**B1**	**1.1**	For comparison leading to correct conclusion	
			or Under H_0, $\overline{X} \sim N\left(210, \frac{3.4^2}{8}\right)$	**M1**	**3.4**		
			$P\left(\overline{X} \leq 207.3\right) = 0.01235$	**A1**	**1.1**	**BC**	
			0.01235< 0.05 so significant	**B1**	**1.1**		
			There is sufficient evidence to reject H_0	**A1**	**2.2b**		
			There is sufficient evidence to conclude that the mean lifetime is less than 210 minutes.	**E1**	**2.4**		
				[5]			

Question			Answer	Marks	AOs	Guidance	
16	**(i)**		Comment about shape of distribution for first graph Comment about shape of distribution for second graph	**B1** **B1** **[2]**	**2.2b** **2.2b**	Comments can be combined e.g Both distributions negatively skewed gets both marks e.g. 1974 distribution has greater spread than 2014 gets both marks	If zero scored, **SC1** for "The 2014 distribution is shifted to the right of the 1974 distribution" oe
16	**(ii)**	(*A*)	Life expectancy went down [between 1974 and 2014] in [at least] one country	**E1** **[1]**	**2.2a**	**NOT** increase in life expectancy is negative	
16	**(ii)**	(*B*)	The box plot is not symmetrical.	**B1** **[1]**	**3.5b**		
16	**(ii)**	(*C*)	Not appropriate with reason	**E1** **[1]**	**2.4**	e.g. [some] values of life expectancy are estimates The values of life expectancy are not available to this level of accuracy	
16	**(ii)**	(*D*)	Comment about life expectancy at birth data for countries and not individual people	**B1** **[1]**	**2.4**		
16	**(iii)**		Use of Q3 + 1.5 × (Q3 - Q1) 15.873 + 1.5(8.9154) = 29.2461 (years) The maximum value is an outlier as 30.742>29.2461.	**M1** **M1** **A1** **[3]**	**1.2** **1.1** **1.1**		

Question			Answer	Marks	AOs	Guidance	
16	(iv)	(*A*)		M1	3.1b	Attempt to estimate change in life expectancy at birth soi.	
			approx 60.8 - 37.5= 23.3 (years)	A1	1.1	FT 'their 37.5 between 35 - 40'	
		(*B*)	Change in life expectancy for Sweden approx 81.9 - 72.5 = 9.4 (years)	A1	1.1	FT 'their 72.5 between 70 - 75'	
		(*C*)	E.g. Countries with a lower life expectancy in 1974 have greater opportunity to increase life expectancy in 2014.	E1	3.2a	OR Countries with a higher life expectancy in 1974 have less opportunity to increase life expectancy in 2014.	
				[4]			
16	(v)	(*A*)	30.98 + 0.67 × 37.4 = 56.0 (years)	M1 A1 [2]	3.4 1.1		
16	(v)	(*B*)	E.g. Large amount of scatter at the lower values [and South Sudan is 37.4].	E1	3.5b	E1 Reason inferred from Fig 16.4	
			E.g. Not having the data value could indicate that there are problems in the country which could mean it does not follow the pattern for other countries	E1 [2]	3.5b	E1 For knowing why data may be missing	
16	(vi)		Correct method	M1	3.1b	e.g. draw "$y = x$" on graph	
			Clearly explained	E1	2.4	e.g. The value on the vertical axis must be lower than the one on the horizontal axis	
			6	A1 [3]	1.1	FT their correct method	

Question	AO1	AO2	AO3(PS)	AO3(M)	Total	LDS
1	3	2			**5**	
2	4				**4**	
3	3				**3**	
4	5				**5**	
5	1			1	**2**	
6 i		1		1	**2**	
6 ii	2				**2**	
7	3		1		**4**	
8		2	1		**3**	
9 i	1	1			**2**	
9 ii		2			**2**	
10		3			**3**	
11		3	1		**4**	
12 i	3	1			**4**	
12 ii	1	1			**2**	
13	4		2		**6**	
14 i	5	2	1		**8**	
14 iiA	2				**2**	
14 iiB		1	1		**2**	
15 i	1		1		**2**	
15 ii	3				**3**	
15 iii				2	**2**	
15 iv	1			2	**3**	
15 v	2	2		1	**5**	
16 i		2			**2**	
16 iiA		1			**1**	
16 iiB				1	**1**	
16 iiC		1			**1**	1
16 iiD		1			**1**	1
16 iii	3				**3**	
16 ivA	1		1		**2**	
16 ivB	1				**1**	
16 ivC			1		**1**	1
16 vA	1			1	**2**	
16 vB				2	**2**	1
16 vi	1	1	1		**3**	
Totals	**51**	**27**	**11**	**11**	**100**	LDS
						4