Oxford Cambridge and RSA

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AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \ldots\ + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \qquad \left(|x|<1,\ \ n \in \mathbb{R}\right)$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \ \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions

1 Simplify $\dfrac{\left(2x^2y\right)^3 \times 4x^3y^5}{2xy^{10}}$. **[2]**

2 Find the coefficient of x^4 in the binomial expansion of $\left(x-3\right)^5$. **[3]**

3 Fig. 3 shows a particle of weight 8 N on a rough horizontal table. The particle is being pulled by a horizontal force of 10 N. It remains at rest in equilibrium.

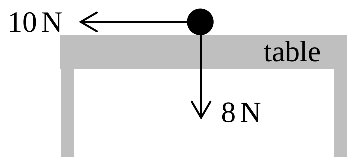

Fig. 3

(i) What information given in the question tells you that the forces shown in Fig. 3 cannot be the only forces acting on the particle? **[1]**

(ii) The only other forces acting on the particle are due to the particle being on the table. State the types of these forces and their magnitudes. **[2]**

4 **(i)** Express x^2+4x+7 in the form $(x+b)^2+c$. **[2]**

(ii) Explain why the minimum point on the curve $y=(x+b)^2+c$ occurs when $x=-b$. **[1]**

5 A particle P moves on a straight line that contains the point O. At time t seconds the displacement of P from O is s metres, where $s=t^3-3t^2+3$.

(i) Determine the times when the particle has zero **velocity**. **[3]**

(ii) Find the distances of P from O at the times when it has zero velocity. **[2]**

6 Two points, A and B, have position vectors $\mathbf{a}=\mathbf{i}-3\mathbf{j}$ and $\mathbf{b}=4\mathbf{i}+3\mathbf{j}$. The point C lies on the line $y=1$. The lengths of the line segments AC and BC are equal. Determine the position vector of C. **[4]**

7 A car is usually driven along the whole of a 5 km stretch of road at a constant speed of $25\,\text{m s}^{-1}$. On one occasion, during a period of 50 seconds, the speed of the car is as shown by the speed-time graph in Fig. 7; the rest of the 5 km is travelled at $25\,\text{m s}^{-1}$.

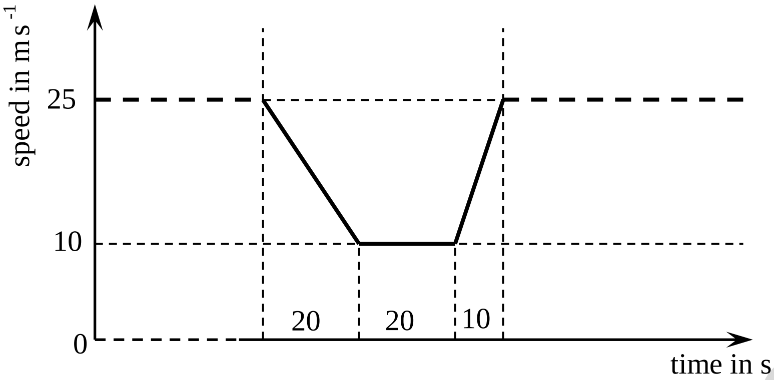

Fig. 7

How much more time than usual did the journey take on this occasion? Show your working clearly. **[4]**

8 A circle has equation $(x-2)^2+(y+3)^2=25$.

(i) Write down

- the radius of the circle,
- the coordinates of the centre of the circle. **[2]**

(ii) Find, in exact form, the coordinates of the points of intersection of the circle with the y-axis. **[3]**

(iii) Show that the point $(1, 2)$ lies outside the circle. **[2]**

(iv) The point $\text{P}(-1, 1)$ lies on the circle. Find the equation of the tangent to the circle at P. **[4]**

9 A biologist is investigating the growth of bacteria in a piece of bread. He believes that the number, N, of bacteria after t hours may be modelled by the relationship $N = A \times 2^{kt}$, where A and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} N$ against t is a straight line.
Give, in terms of A and k,
- the gradient of the line
- the intercept on the vertical axis. **[4]**

The biologist measures the number of bacteria at regular intervals over 22 hours and plots a graph of $\log_{10} N$ against t. He finds that the graph is approximately a straight line with gradient 0.20; the line crosses the vertical axis at 2.0.

(ii) Find the values of A and k. **[2]**

(iii) Use the model to predict the number of bacteria after 24 hours. **[1]**

(iv) Give a reason why the model may not be appropriate for large values of t. **[1]**

10 **(i)** Sketch the graph of $y = \frac{1}{x} + a$, where a is a positive constant.
- State the equations of the horizontal and vertical asymptotes.
- Give the coordinates of any points where the graph crosses the axes. **[4]**

(ii) Find the equation of the normal to the curve $y = \frac{1}{x} + 2$ at the point where $x = 2$. **[5]**

(iii) Find the coordinates of the point where this normal meets the curve again. **[3]**

11 **In this question you must show detailed reasoning.**
Determine for what values of k the graphs $y = 2x^2 - kx$ and $y = x^2 - k$ intersect. **[6]**

12 A small package hangs from a balloon by means of a light inelastic string. The string is always vertical. The mass of the package is 15 kg.

Catherine initially models the situation by assuming that there is no air resistance to the motion of the package. Use Catherine's model to calculate the tension in the string if

(i) the package is held at rest by the tension in the string, **[1]**

(ii) the package is instantaneously at rest and accelerating **upwards** at $2\,\text{m}\,\text{s}^{-2}$, **[2]**

(iii) the package is moving **downwards** at $3\,\text{m}\,\text{s}^{-1}$ and accelerating **upwards** at $2\,\text{m}\,\text{s}^{-2}$. **[1]**

Catherine now carries out an experiment to find the magnitude of the air resistance on the package when it is moving. At a time when the package is accelerating **downwards** at $1.5\,\text{m}\,\text{s}^{-2}$, she finds that the tension in the string is 140 N.

(iv) Calculate the magnitude of the air resistance at that time. Give, with a reason, the direction of motion of the package. **[5]**

END OF QUESTION PAPER