OCR
Oxford Cambridge and RSA

A Level Mathematics A
H240/03 Pure Mathematics and Mechanics
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r} \quad \text{for } |r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 \, a^{n-1}b + {}^nC_2 \, a^{n-2}b^2 + \ldots + {}^nC_r \, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \left(|x| < 1, \ n \in \mathbb{R}\right)$

Differentiation

f(x)	f$'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

Quotient rule $\ y = \dfrac{u}{v}, \ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Integration

$\displaystyle\int \dfrac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\displaystyle\int f'(x)\big(f(x)\big)^n \,dx = \dfrac{1}{n+1}\big(f(x)\big)^{n+1} + c$

Integration by parts $\ \displaystyle\int u \dfrac{dv}{dx}\,dx = uv - \int v \dfrac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B) \qquad \textbf{or} \qquad P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$$\sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2} \ \textbf{ or } \ \sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n,\ p)$ then $P(X = x) = \dbinom{n}{x} p^x (1 - p)^{n-x}$, Mean of X is np, Variance of X is $np(1 - p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N\left(\mu,\ \sigma^2\right)$ then $\bar{X} \sim N\left(\mu,\ \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

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Section A: Pure Mathematics
Answer **all** the questions

1   **(i)** If $|x| = 3$, find the possible values of $|2x-1|$.   **[3]**

   (ii) Find the set of values of x for which $|2x-1| > x+1$. Give your answer in set notation.   **[4]**

2   **(i)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\, dx.$$ **[3]**

   (ii) Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(i)**.   **[1]**

3   **In this question you must show detailed reasoning.**

   Given that $5\sin 2x = 3\cos x$, where $0° < x < 90°$, find the exact value of $\sin x$.   **[4]**

4   Show that, for a small angle θ, where θ is in radians,

$$1 + \cos\theta - 3\cos^2\theta \approx -1 + \tfrac{5}{2}\theta^2.$$

   [4]

5   **(i)** Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x.   **[3]**

   (ii) Given that the expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is

$$1 + x - \tfrac{2}{9}x^2 + \ldots$$

   find the possible values of p and q.   **[5]**

6   A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant.
   Given that the curve has a minimum point when $x = -2$
   - find the value of k,
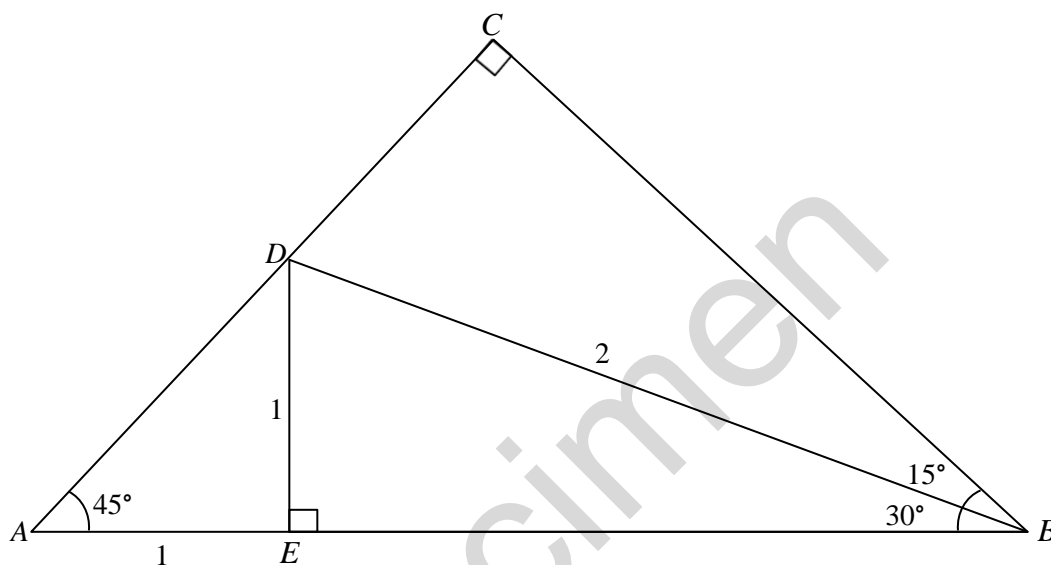   - show that the curve has a point of inflection which is not a stationary point.

   [7]

7 **(i)** Find $\int 5x^3 \sqrt{x^2+1}\ dx$. **[5]**

(ii) Find $\int \theta \tan^2 \theta\,d\theta$.

You may use the result $\int \tan \theta\,d\theta = \ln|\sec \theta| + c$. **[5]**

**8** **In this question you must show detailed reasoning.**

The diagram shows triangle ABC. The angles CAB and ABC are each $45°$, and angle $ACB = 90°$. The points D and E lie on AC and AB respectively, such that $AE = DE = 1$, $DB = 2$ and angle $BED = 90°$. Angle $EBD = 30°$ and angle $DBC = 15°$.

(i) Show that $BC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

(ii) By considering triangle BCD, show that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

Section B: Mechanics
Answer **all** the questions

9   Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.
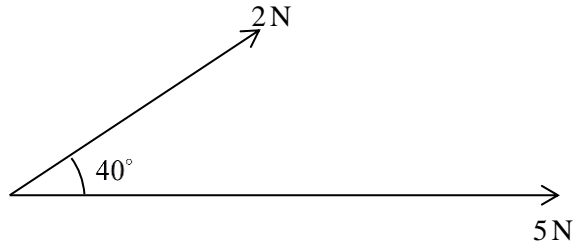

   (i) Calculate the magnitude of the resultant force on the particle. **[3]**

   (ii) Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

10   A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal. The body is held at rest on the plane by the action of a force of magnitude P N acting up the plane in a direction parallel to a line of greatest slope of the plane. The coefficient of friction between the body and the plane is μ.

   (i) When $P = 100$, the body is on the point of sliding down the plane.
   Show that $g \sin \alpha = g \mu \cos \alpha + 5$. **[4]**

   (ii) When P is increased to 150, the body is on the point of sliding up the plane. Using this and your answer to part **(i)**, find an expression for α in terms of g. **[3]**

11   In this question the unit vectors **i** and **j** are in the directions east and north respectively.

   A particle of mass 0.12 kg is moving so that its position vector **r** metres at time t seconds is given by
   $\mathbf{r} = 2t^3 \mathbf{i} + \left(5t^2 - 4t\right)\mathbf{j}$.

   (i) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately $044°$. **[3]**

   (ii) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

   (iii) Determine the times at which the particle is moving on a bearing of $045°$. **[2]**

12 A girl is practising netball. She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop. The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude $U\,\text{m s}^{-1}$.
- The angle of projection is $40°$.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.
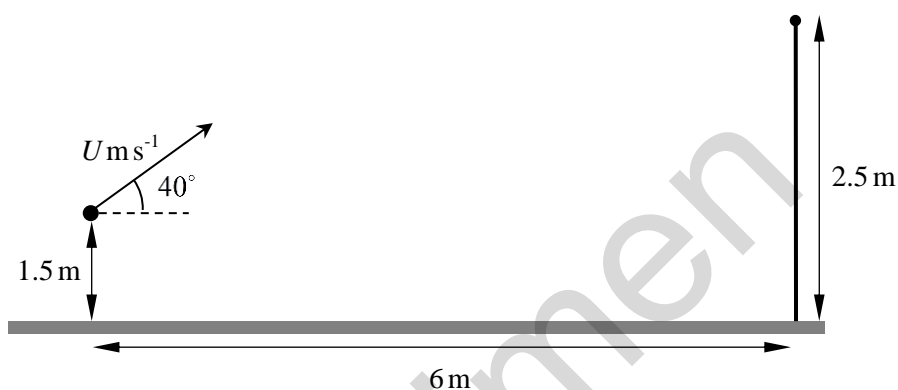
 (i) For $U = 10$, find

   (a) the greatest height above the ground reached by the ball, **[5]**

   (b) the distance between the ball and the hoop when the ball is vertically above the hoop. **[4]**

 (ii) Calculate the value of U which allows her to hit the hoop. **[3]**

 (iii) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

 (iv) Suggest one improvement that might be made to this model. **[1]**

13 Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal. Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal. The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P. The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth. Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.
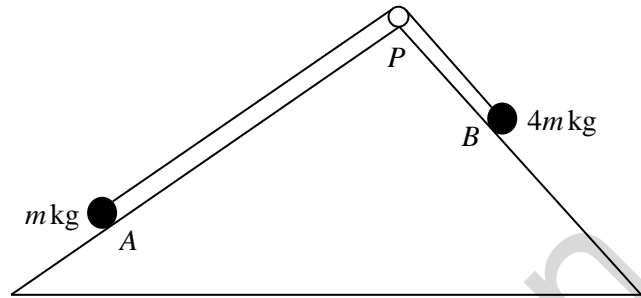
(i) Show that when A is released it accelerates towards the pulley at $\dfrac{7g}{15}$ m s^{-2}. **[6]**

(ii) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is

$$\sqrt{\dfrac{7g}{30}} \text{ m s}^{-1}.$$ **[2]**

14 A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder is inclined at an angle of $45°$ to the horizontal. A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$. The system is in limiting equilibrium.
Find x. **[8]**

END OF QUESTION PAPER