OCR
Oxford Cambridge and RSA

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AS Level Mathematics A

H230/01 Pure Mathematics and Statistics
Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \ \text{ or } \ \sqrt{\frac{\Sigma f\left(x-\bar{x}\right)^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}\left(u+v\right)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions

1 It is given that $\mathrm{f}(x) = 6x^3 - 5x$. Find

(i) $\mathrm{f}'(x)$, **[2]**

(ii) $\mathrm{f}''(2)$. **[2]**

2 Points *A* and *B* have coordinates (3, 0) and (9, 8) respectively. The line *AB* is a diameter of a circle.

(i) Find the coordinates of the centre of the circle. **[2]**

(ii) Find the equation of the tangent to the circle at the point *B*. **[3]**

3 The points *P*, *Q* and *R* have coordinates $(-1, 6)$, $(2, 10)$ and $(11, 1)$ respectively. Find the angle *PRQ*. **[4]**

4 The curve $y = 2x^3 + 3x^2 - kx + 4$ has a stationary point where $x = 2$.

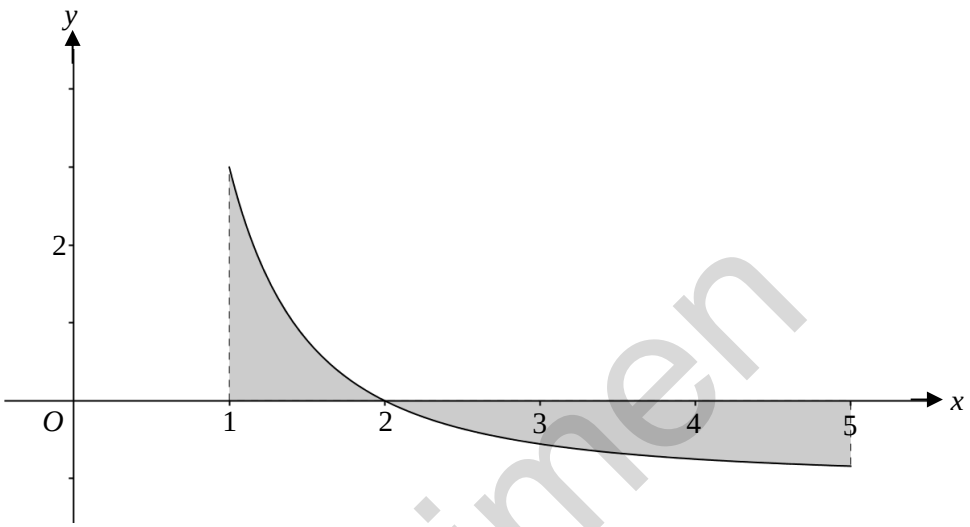
(i) Determine the value of the constant *k*. **[5]**

(ii) Determine whether this stationary point is a maximum or a minimum point. **[2]**

5 **(i)** Find $\int\left(x^3-6x\right)\mathrm{d}x$. **[3]**

(ii) **(a)** Find $\int\left(\frac{4}{x^2}-1\right)\mathrm{d}x$. **[3]**

(b) The diagram below shows part of the curve $y=\frac{4}{x^2}-1$. The curve crosses the x-axis at (2, 0). The shaded region is bounded by the curve, the x-axis, and the lines $x=1$ and $x=5$.



Calculate the area of the shaded region. **[3]**

6 **In this question you must show detailed reasoning.**

The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x)=4x^3+4x^2+7x-5$.

(i) Show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. **[2]**

(ii) Hence solve the equation $4\sin^3\theta+4\sin^2\theta+7\sin\theta-5=0$ for $0^\circ\leq\theta\leq360^\circ$. **[7]**

7 **(i)** Sketch the curve $y=2x^2-x-3$. **[3]**

(ii) Hence, or otherwise, solve $2x^2-x-3<0$. **[2]**

(iii) Given that the equation $2x^2-x-3=k$ has no real roots, find the set of possible values of k. **[3]**

Section B: Statistics

Answer **all** the questions

8 A club secretary wishes to survey a sample of members of his club. He uses all members present at a particular meeting as his sample.

(i) Explain why this sample is likely to be biased. **[1]**

Later the secretary decides to choose a random sample of members. The club has 253 members and the secretary numbers the members from 1 to 253. He then generates random 3-digit numbers on his calculator. The first six random numbers generated are 156, 965, 248, 156, 073 and 181. The secretary uses each number, where possible, as the number of a member in the sample.

(ii) Find possible numbers for the first four members in the sample. **[2]**

9 The probability distribution of a random variable X is given in the table.

x	1	2	3
$\mathrm{P}(X = x)$	0.6	0.3	0.1

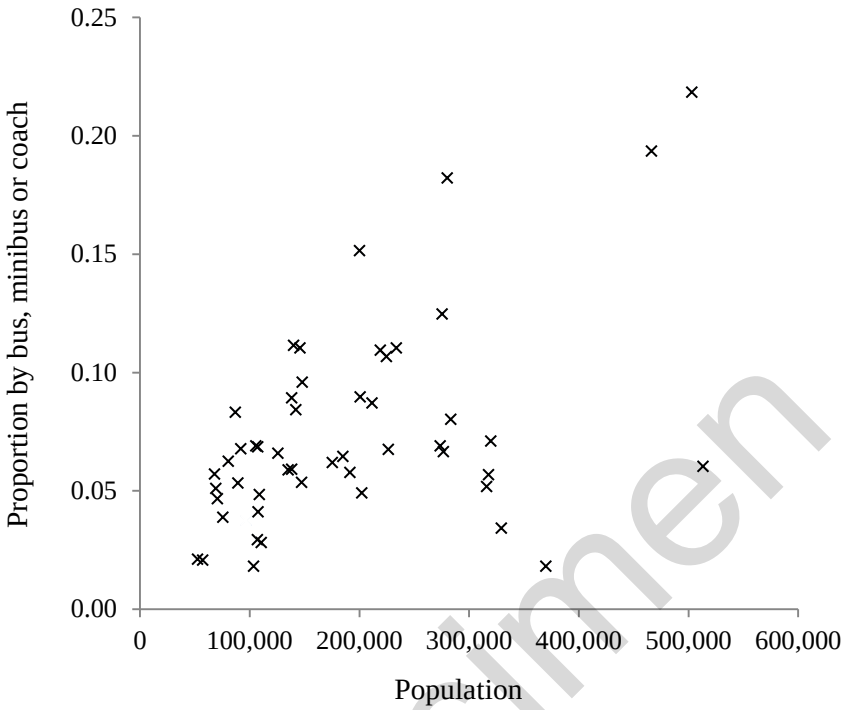
Two values of X are chosen at random. Find the probability that the second value is greater than the first. **[3]**

10 **(i)** Write down and simplify the first four terms in the expansion of $(x+y)^7$, in ascending powers of x. **[2]**

(ii) Given that the terms in x^2y^5 and x^3y^4 in this expansion are equal, find the value of $\frac{x}{y}$. **[2]**

(iii) A hospital consultant has seven appointments every day. The number of these appointments which start late on a randomly chosen day is denoted by L. The variable L is modelled by the distribution $\mathrm{B}\left(7, \frac{3}{8}\right)$. Show that, in this model, the hospital consultant is equally likely to have two appointments start late or three appointments start late. **[3]**

11 The scatter diagram below shows data taken from the 2011 UK census. The scatter diagram shows, for each of the Local Authorities in the North East and North West regions, the total population of the Local Authority and the proportion of its workforce that travel to work by bus, minibus or coach.



(i) Samuel suggests that, with a few exceptions, the data points in the diagram show that Local Authorities with larger populations generally have higher proportions of workers travelling by bus, minibus or coach. On the diagram in the Printed Answer Booklet draw a ring around each of the data points that Samuel might regard as an exception. **[1]**

(ii) Jasper suggests that it is possible to separate these Local Authorities into more than one group with different relationships between population and proportion travelling to work by bus, minibus or coach. Discuss Jasper's suggestion, referring to the data and to how differences between the Local Authorities could explain the patterns seen in the diagram. **[3]**

12 It is known that under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year. In a trial of a new treatment, 450 patients with this disease were selected and the number, X, that experienced side effects within one year was noted.
It was found that 51 of the 450 patients experienced side effects within one year.

(i) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment. **[7]**

(ii) It was later discovered that all 450 patients selected for the trial were treated in the same hospital. Comment on the validity of the model used in part **(i)**. **[1]**

13 Clara used some data from the 2011 UK census to summarise information on carbon emissions due to travel to work, in two Local Authorities. Her results are shown below.

	Method of travel to work	Individual motorised transport	Shared motorised transport	Public transport	No motorised transport	
	Carbon emissions category	High	Medium	Low	None	Total
Local Authority A	Number of workers	174 374	42 112	61 483	76 024	353 993
	Percentage of workers	49.3	11.9	17.4	21.5	100
Local Authority B	Number of workers	39 433	9944	4614	16 232	70 223
	Percentage of workers	56.2	14.2	6.6	23.1	100

(i) Clara calculated the values for the column headed "shared motorised transport" by doubling the value in the "passenger in a car or van" column of the original data set. Explain what assumption she has made and what other adjustment would need to be made to the data to take account of this. **[2]**

(ii) Clara suggests that the average carbon emissions per worker due to travelling to work is larger in region B than in region A.

(a) Use data from the table to support Clara's suggestion. **[1]**

(b) Use data from the table to argue against Clara's suggestion. **[1]**

END OF QUESTION PAPER